$$\int_0^{\pi/2} \cos^n x\,dx$$

let $I_n = \int_0^{\pi/2} \cos^n x\,dx$

we have, $I_n = \frac{\cos^{n-1}x\sin x}{n} + \frac{(n-1)}{n} I_{n-2}$ $\left[\because \cos\frac{\pi}{2} = 0, \sin 0 = 0\right]$

$$I_n = \frac{(n-1)}{n} I_{n-2}$$

from above eqn, we have

$$I_{n-2} = \frac{n-3}{n-2} I_{n-4}$$

$$I_{n-4} = \frac{n-5}{n-4} I_{n-6}$$

$$I_{n-6} = \frac{n-7}{n-6} I_{n-8}$$

. . . .

$$I_3 = \frac{2}{3} I_1$$

$$I_2 = \frac{1}{2} I_0$$

$$\therefore I_n = \frac{(n-1)}{n} I_{n-2}$$

$$= \frac{(n-1)}{n}\cdot\frac{(n-3)}{(n-2)} I_{n-4}$$

$$= \frac{(n-1)}{n}\frac{(n-3)}{(n-2)}\frac{(n-5)}{(n-4)} I_{n-6}$$

— — — —

$$I_n = \begin{cases} \frac{(n-1)}{n}\cdot\frac{(n-3)}{(n-2)}\cdot\frac{(n-5)}{(n-4)}\cdots\frac{2}{3} I_1 & \text{if n is odd} \\ \frac{(n-1)}{n}\cdot\frac{(n-3)}{(n-2)}\frac{(n-5)}{(n-4)}\cdots\frac{1}{2} I_0 & \text{if n is even} \end{cases}$$

$$I_1 = \int_0^{\pi/2} \cos^1 x\,dx = \sin x\Big|_0^{\pi/2} = 1 - 0 = 1$$

$$\& \; I_0 = \int_0^{\pi/2} \cos^0 x\,dx = \int_0^{\pi/2} dx = x\Big|_0^{\pi/2} = \pi/2$$

$$I_n = \begin{cases} \frac{(n-1)}{n}\cdot\frac{(n-3)}{(n-2)}\cdot\frac{(n-5)}{(n-4)}\cdots\frac{2}{3} & \text{if n is odd} \\ \frac{(n-1)}{n}\frac{(n-3)}{(n-2)}\frac{(n-5)}{(n-4)}\cdot\frac{1}{2}\frac{\pi}{2} & \text{if n is even} \end{cases}$$